

The Mandelstam-Terning Line Integral in Unparticle Physics

A Reply to Galloway, Martin and Stancato

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We show that the path ordered Wilson line integral used to make a nonlocal action gauge invariant is mathematically inconsistent. We also show that it can lead to reasonable gauge field vertexes by the use of a second mathematically unjustifiable procedure.

INTRODUCTION

There have been attempts to make the non local unparticle action introduced by Georgi [1] [2] gauge invariant [3] [4] by introducing in the action the path ordered Wilson line

$$W_P(y, x) = P \exp \left[-ig \int_x^y A_\mu(w) dw^\mu \right] \quad (1)$$

where here

$$A_\mu(w) = T^a A_\mu^a(w) \quad (2)$$

and W_P is assumed to satisfy the condition introduced by Mandelstam [5]

$$\frac{\partial}{\partial y^\mu} W_P(y, x) = -ig A_\mu(y) W_P(y, x) \quad (3)$$

We pointed out in [7] that W_P is not well defined if the path between x and y is not specified. We took as the path a straight line and derived some rather complicated gauge-particle vertexes. Galloway, Martin and Stancato [GMS] [6] have shown that W_P with condition (3) does give vertexes that satisfy the “Minimal Coupling” requirement, that is, when the unparticle dimension is an integer, the coupling is what one would expect from simply replacing in the action $i\partial_\mu$ by $i\partial_\mu - gA_\mu$. They also point out that for integer dimension other than 1, the straight line method of Ref. [7] gives results that do not satisfy the Minimal Coupling requirement. In a later work, [8] we showed that $i\partial_\mu - gA_\mu$ can be defined as a differential-integral operator, can be used in unparticle actions of arbitrary dimension and gives vertexes that automatically satisfy the Minimal Coupling requirement.

Here we will show that the Wilson line with condition (3) is mathematically very shaky. We will also show why nevertheless it does seem to give reasonable vertexes.

THE MANDELSTAM CONDITION THEOREM

Theorem: If the Wilson line operator W_P satisfies condition (3) then the field strength $F_{\mu\nu}$ vanishes and the gauge field A_μ is gauge equivalent to zero.

Proof: The second partial derivative of W_P gives, using condition (3),

$$\frac{\partial}{\partial y^\nu} \frac{\partial}{\partial y^\mu} W_P(y, x) = -ig \frac{\partial}{\partial y^\nu} A_\mu(y) W_P(y, x) - g^2 A_\mu(y) A_\nu(y) W_P(y, x) \quad (4)$$

But, second derivatives commute, so

$$\begin{aligned} 0 &= \left(\frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} - \frac{\partial}{\partial y^\nu} \frac{\partial}{\partial y^\mu} \right) W_P(y, x) \\ &= -ig F_{\mu\nu}(y) W_P(y, x) \end{aligned} \quad (5)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \quad (6)$$

is the field strength. However, W_P is unitary, therefore

$$F_{\mu\nu} = 0 \quad (7)$$

This is the condition that A_μ be gauge equivalent to zero. It is possible to find the transformation that takes A_μ into zero. Eq. (3) can also be written as

$$\frac{\partial}{\partial x^\mu} W_P(y, x) = +ig W_P(y, x) A_\mu(x) \quad (8)$$

Under a gauge transformation V ,

$$\begin{aligned} \Phi_u &\rightarrow \Phi'_u = V \Phi_u \\ A_\mu &\rightarrow A'_\mu = V A_\mu V^\dagger + \frac{i}{g} (\partial_\mu V) V^\dagger \\ F_{\mu\nu} &\rightarrow F'_{\mu\nu} = V F_{\mu\nu} V^\dagger \end{aligned} \quad (9)$$

We have

$$(i\partial_\mu - gA_\mu) \Phi_u \rightarrow (i\partial_\mu - gA'_\mu) \Phi'_u = V (i\partial_\mu - gA_\mu) \Phi_u \quad (10)$$

If we take for the gauge transformation the unitary operator

$$V(x) = W_P(0, x) \quad (11)$$

then Eq. (8) leads to

$$\partial_\mu V = ig V A_\mu \quad (12)$$

which makes the A'_μ of Eq. (9) equal to zero. qed.

VERTEXES

The question now arises, if the gauge field is actually zero, why does the Terning method lead to reasonable vertexes? Basically this is a case of two wrongs making a right. Introducing the Wilson line of Eq. (1) into the unparticle action leads to an action that can be expanded in a power series in the A_μ field:

$$I = \sum_{n=0}^{\infty} \int d^4x d^4y \Phi_u^\dagger(y) \prod_{k=1}^n \int d^4z_k A_{\mu_k}(z_k) K_k(x, y, \{z_i\}) \Phi_u(x) \quad (13)$$

The vertexes are found by taking functional derivatives with respect to the A_μ field, using

$$\frac{\delta A_\alpha^a(z)}{\delta A_\beta^b(w)} = \delta^{ab} \delta_\alpha^\beta \delta^4(z - w) \quad (14)$$

However, as a pure gauge field A_α^a must be written in terms of a field Λ^a where, with

$$\Lambda = \Lambda^a T^a \quad (15)$$

and

$$V = e^{-ig\Lambda} \quad (16)$$

we get

$$\begin{aligned} A_\alpha &= \frac{i}{g} (\partial_\alpha V) V^\dagger \\ &= \int_0^1 e^{-itg\Lambda} (\partial_\alpha \Lambda) e^{itg\Lambda} dt \end{aligned} \quad (17)$$

For the simple case of $U(1)$, or for more general gauge groups with infinitesimal Λ ,

$$A_\alpha = \partial_\alpha \Lambda \quad (18)$$

and although we can write

$$\frac{\delta \partial_\alpha \Lambda^a(z)}{\delta \Lambda^b(x)} = \delta^{ab} \frac{\partial}{\partial z^\alpha} \delta^4(x - z) \quad (19)$$

there is no way to convert the $\partial/\partial z^\alpha$ into a δ_α^β and the functional derivative of Eq. (14) cannot be carried out.

What is actually being done, is to replace, by hand, the actual $\partial_\mu \Lambda^a$ dependent terms provided by the Wilson line factor with non pure gauge A_μ^a factors and then to differentiate. This should give the correct vertexes, if, as one expects, the Wilson line factor puts the pure gauge factors exactly where the principal of minimal coupling says they should be. This is certainly true for integer unparticle dimension, as GMS have shown.

CONCLUSIONS

We have shown that the Mandelstam assumption about the Wilson line operator implies that the gauge fields are gauge equivalent to zero. We show that nevertheless the resulting unparticle action can be converted, by hand, to a form that gives the correct vertexes.

We would like to point out however that the mechanism that gives rise to unparticles is not precisely known. It is conceivable that the unparticle effective action might not actually obey the minimal coupling principle.

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